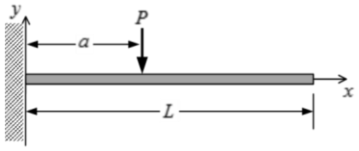
Four Statically-Determinate Problems

SD-1



for $0 \le x \le a \quad v(x) = \dfrac{P}{6EI}\left(x^3 - 3ax^2\right)$

for $a \le x \le L \quad v(x) = \dfrac{P}{6EI}\left(a^3 - 3a^2x\right)$

1. Input

$$q(x) = P\delta\left(x-a\right), \quad \tilde{q} = P$$

2. Elastic creep solution

$$v^e(x) = \begin{cases} \dfrac{P}{6EI}x^2\left(x-3a\right) & \text{at} \quad 0 \le x \le a \\ \dfrac{P}{6EI}a^2\left(a-3x\right) & \text{at} \quad a \le x \le L \end{cases}$$

3. Confirm that $M_z^e(x) = EIv^{e\prime\prime}(x)$ is not a function of E. → OK for correspondence theorem.

4. Representative deflection: $\bar{x} = L$ (Choose the max deflection location)

$$g_v(x) = \begin{cases} \dfrac{x^2\left(x-3a\right)}{6} & \text{at} \quad 0 \le x \le a \\ \dfrac{a^2(a-3x)}{6} & \text{at} \quad a \le x \le L \end{cases}$$

$$g_v(\bar{x} = L) = \frac{a^2\left(a-3L\right)}{6}, \qquad \tilde{v}^e = \frac{P}{6EI}a^2\left(a-3L\right)$$

5. Shape functions

$$f_v(x) = \begin{cases} \dfrac{x^2\left(x-3a\right)}{a^2\left(a-3L\right)} & \text{at} \quad 0 \le x \le a \\ \dfrac{a-3x}{a-3L} & \text{at} \quad a \le x \le L \end{cases}$$

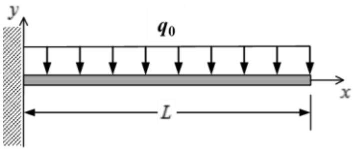
6. Generalized creep

$$\tilde{v}^{ve} = \frac{a^2\left(a-3L\right)}{6I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dP(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{v}^{ve} = \frac{a^2\left(a-3L\right)}{6I}J(t)P$$

7. Generalized relaxation

$$\tilde{q}^{ve} = P^{ve} = \frac{6I}{a^2\left(a-3L\right)}\int_{0^-}^{t} Y\left(t-\tau\right)\frac{d\tilde{v}(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{q}^{ve} = P^{ve} = \frac{6I}{a^2\left(a-3L\right)}Y(t)\tilde{v}$$

SD-2



$$v(x) = -\frac{q_0}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$

1. Input

$$q(x) = q_0, \quad \tilde{q} = q_0$$

2. Elastic creep solution

$$v^e(x) = -\frac{q_0}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$

3. Confirm that $M_z^e(x) = EIv^{e''}(x)$ is not a function of E. → OK for correspondence theorem.

4. Representative deflection: $\bar{x} = L$ (Choose the max deflection location)

$$g_v(x) = -\frac{1}{24}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$

$$g_v(\bar{x} = L) = -\frac{L^4}{8}, \qquad \tilde{v}^e = -\frac{q_0 L^4}{8EI}$$

5. Shape functions

$$f_v(x) = \frac{1}{3L^4}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$

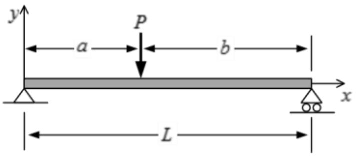
6. Generalized creep

$$\tilde{v}^{ve} = -\frac{L^4}{8I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dq_0(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{v}^{ve} = -\frac{L^4}{8I}J(t)q_0$$

7. Generalized relaxation

$$\tilde{q}^{ve} = -\frac{8I}{L^4}\int_{0^-}^{t} Y\left(t-\tau\right)\frac{d\tilde{v}(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{q}^{ve} = -\frac{8I}{L^4}Y(t)\tilde{v}$$

SD-3



for $0 \le x \le a$ $\quad v(x) = \dfrac{P}{6EI}\dfrac{L-a}{L}\left(x^3 - a(2L-a)x\right)$

for $a \le x \le L$ $\quad v(x) = \dfrac{P}{6EI}\dfrac{L-a}{L}\left(x^3 - a(2L-a)x - \dfrac{L}{L-a}(x-a)^3\right)$

1. Input

$$q(x) = P\delta(x-a), \quad \tilde{q} = P$$

2. Elastic creep solution

$$v^e(x) = \begin{cases} \dfrac{P}{6EI}\dfrac{L-a}{L}\left(x^3 - a(2L-a)x\right) & \text{at} \quad 0 \le x \le a \\ \dfrac{P}{6EI}\dfrac{L-a}{L}\left(x^3 - a(2L-a)x - \dfrac{L}{L-a}(x-a)^3\right) & \text{at} \quad a \le x \le L \end{cases}$$

3. Confirm that $M_z^e(x) = EIv^{e\prime\prime}(x)$ is not a function of E. → OK for correspondence theorem.

4. Representative deflection: $\bar{x} = a$ (Hard to find max location, choose simplicity)

$$g_v(x) = \begin{cases} \dfrac{L-a}{6L}\left(x^3 - a(2L-a)x\right) & \text{at} \quad 0 \le x \le a \\ \dfrac{L-a}{6L}\left(x^3 - a(2L-a)x - \dfrac{L}{L-a}(x-a)^3\right) & \text{at} \quad a \le x \le L \end{cases}$$

$$g_v(\bar{x} = a) = -\frac{a^2(L-a)^2}{3L}, \qquad \tilde{v}^e = -\frac{Pa^2(L-a)^2}{3EIL}$$

5. Shape functions

$$f_v(x) = \begin{cases} -\dfrac{1}{2a^2(L-a)}\left(x^3 - a(2L-a)x\right) & \text{at} \quad 0 \le x \le a \\ -\dfrac{1}{2a^2(L-a)}\left(x^3 - a(2L-a)x - \dfrac{L}{L-a}(x-a)^3\right) & \text{at} \quad a \le x \le L \end{cases}$$

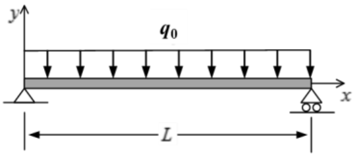
6. Generalized creep

$$\tilde{v}^{ve} = -\frac{a^2(L-a)^2}{3IL}\int_{0^-}^{t} J(t-\tau)\frac{dP(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{v}^{ve} = -\frac{a^2(L-a)^2}{3IL}J(t)P$$

7. Generalized relaxation

$$\tilde{q}^{ve} = P^{ve} = -\frac{3IL}{a^2(L-a)^2}\int_{0^-}^{t} Y(t-\tau)\frac{d\tilde{v}(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{q}^{ve} = P^{ve} = -\frac{3IL}{a^2(L-a)^2}Y(t)\tilde{v}$$

SD-4



$$v(x) = -\frac{q_0}{24EI}\left(x^4 - 2Lx^3 + L^3 x\right)$$

1. Input

$$q(x) = q_0, \quad \tilde{q} = q_0$$

2. Elastic creep solution

$$v^e(x) = -\frac{q_0}{24EI}\left(x^4 - 2Lx^3 + L^3 x\right)$$

3. Confirm that $M_z^e(x) = EIv^{e\prime\prime}(x)$ is not a function of E. → OK for correspondence theorem.

4. Representative deflection: $\bar{x} = L/2$ (Choose the max deflection location)

$$g_v(x) = -\frac{1}{24}\left(x^4 - 2Lx^3 + L^3 x\right)$$

$$g_v(\bar{x} = \frac{L}{2}) = -\frac{5L^4}{384}, \qquad \tilde{v}^e = -\frac{5q_0 L^4}{384EI}$$

5. Shape functions

$$f_v(x) = \frac{16}{5L^4}\left(x^4 - 2Lx^3 + L^3 x\right)$$

6. Generalized creep

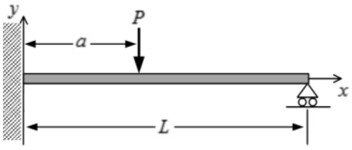
$$\tilde{v}^{ve} = -\frac{5L^4}{384I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dq_0(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{v}^{ve} = -\frac{5L^4}{384I}J(t)q_0$$

7. Generalized relaxation

$$\tilde{q}^{ve} = -\frac{384I}{5L^4}\int_{0^-}^{t} Y\left(t-\tau\right)\frac{d\tilde{v}(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{q}^{ve} = -\frac{384I}{5L^4}Y(t)\tilde{v}$$

Four Statically-Indeterminate Problems

SI-1



for $0 \le x \le a$ $\quad v(x) = \dfrac{P}{6EI}\left[x^3 - 3ax^2 - \dfrac{a^2(3L-a)}{2L^3}\left(x^3 - 3Lx^2\right)\right]$

for $a \le x \le L$ $\quad v(x) = \dfrac{P}{6EI}\left[a^3 - 3a^2x - \dfrac{a^2(3L-a)}{2L^3}\left(x^3 - 3Lx^2\right)\right]$

1. Input

$$q(x) = P\delta\left(x-a\right), \quad \tilde{q} = P$$

2. Elastic creep solution

$$v^e(x) = \begin{cases} \dfrac{P}{6EI}\left[x^3 - 3ax^2 - \dfrac{a^2(3L-a)}{2L^3}\left(x^3 - 3Lx^2\right)\right] & \text{at } \ 0 \le x \le a \\ \dfrac{P}{6EI}\left[a^3 - 3a^2x - \dfrac{a^2(3L-a)}{2L^3}\left(x^3 - 3Lx^2\right)\right] & \text{at } \ a \le x \le L \end{cases}$$

3. Confirm that $M_z^e(x) = EIv^{e\prime\prime}(x)$ is not a function of E. → OK for correspondence theorem.

4. Representative deflection: $\bar{x} = a$ (Hard to find max location, choose simplicity)

$$g_v(x) = \begin{cases} \dfrac{1}{6}\left[x^3 - 3ax^2 - \dfrac{a^2(3L-a)}{2L^3}\left(x^3 - 3Lx^2\right)\right] & \text{at } \ 0 \le x \le a \\ \dfrac{1}{6}\left[a^3 - 3a^2x - \dfrac{a^2(3L-a)}{2L^3}\left(x^3 - 3Lx^2\right)\right] & \text{at } \ a \le x \le L \end{cases}$$

$$g_v(\bar{x} = a) = -\frac{a^3(L-a)(4L-a)}{12L^3}, \qquad \tilde{v}^e = -\frac{Pa^3(L-a)(4L-a)}{12EIL^3}$$

5. Shape functions

$$f_v(x) = \begin{cases} -\dfrac{2L^3}{a^3(L-a)(4L-a)}\left[x^3 - 3ax^2 - \dfrac{a^2(3L-a)}{2L^3}\left(x^3 - 3Lx^2\right)\right] & \text{at } \ 0 \le x \le a \\ -\dfrac{2L^3}{a^3(L-a)(4L-a)}\left[a^3 - 3a^2x - \dfrac{a^2(3L-a)}{2L^3}\left(x^3 - 3Lx^2\right)\right] & \text{at } \ a \le x \le L \end{cases}$$

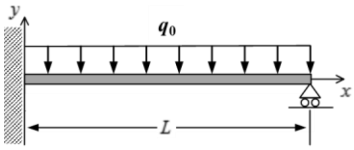
6. Generalized creep

$$\tilde{v}^{ve} = -\frac{a^3(L-a)(4L-a)}{12IL^3}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dP(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{v}^{ve} = -\frac{a^3(L-a)(4L-a)}{12IL^3}J(t)P$$

7. Generalized relaxation

$$\tilde{q}^{ve} = -\frac{12IL^3}{a^3(L-a)(4L-a)}\int_{0^-}^{t} Y\left(t-\tau\right)\frac{d\tilde{v}(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{q}^{ve} = -\frac{12IL^3}{a^3(L-a)(4L-a)}Y(t)\tilde{v}$$

SI-2



$$v(x) = -\frac{q_0}{48EI}\left(2x^4 - 5Lx^3 + 3L^2x^2\right)$$

1. Input

$$q(x) = q_0, \quad \tilde{q} = q_0$$

2. Elastic creep solution

$$v^e(x) = -\frac{q_0}{48EI}\left(2x^4 - 5Lx^3 + 3L^2x^2\right)$$

3. Confirm that $M_z^e(x) = EIv^{e''}(x)$ is not a function of E. → OK for correspondence theorem.

4. Representative deflection: $\bar{x} = L/2$ (simplicity)

$$g_v(x) = -\frac{1}{48}\left(2x^4 - 5Lx^3 + 3L^2x^2\right)$$

$$g_v(\bar{x} = \frac{L}{2}) = -\frac{L^4}{192}, \qquad \tilde{v}^e = -\frac{q_0 L^4}{192EI}$$

5. Shape functions

$$f_v(x) = \frac{4}{L^4}\left(2x^4 - 5Lx^3 + 3L^2x^2\right)$$

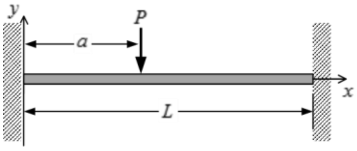
6. Generalized creep

$$\tilde{v}^{ve} = -\frac{L^4}{192I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dq_0(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{v}^{ve} = -\frac{L^4}{192I}J(t)q_0$$

7. Generalized relaxation

$$\tilde{q}^{ve} = -\frac{192I}{L^4}\int_{0^-}^{t} Y\left(t-\tau\right)\frac{d\tilde{v}(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{q}^{ve} = -\frac{192I}{L^4}Y(t)\tilde{v}$$

SI-3



for $0 \le x \le a \quad v(x)=\frac{P}{6EI}\left[x^3-3ax^2-\frac{a^2(3L-2a)}{L^3}\left(x^3-3Lx^2\right)-\frac{3a^2(L-a)}{L^2}x^2\right]$

for $a \le x \le L \quad v(x)=\frac{P}{6EI}\left[a^3-3a^2x-\frac{a^2(3L-2a)}{L^3}\left(x^3-3Lx^2\right)-\frac{3a^2(L-a)}{L^2}x^2\right]$

1. Input

$$q(x)=P\delta\left(x-a\right),\quad \tilde{q}=P$$

2. Elastic creep solution

$$v^e(x)=\begin{cases}\dfrac{P}{6EI}\left[x^3-3ax^2-\dfrac{a^2(3L-2a)}{L^3}\left(x^3-3Lx^2\right)-\dfrac{3a^2(L-a)}{L^2}x^2\right] & \text{at} \quad 0\le x\le a\\ \dfrac{P}{6EI}\left[a^3-3a^2x-\dfrac{a^2(3L-2a)}{L^3}\left(x^3-3Lx^2\right)-\dfrac{3a^2(L-a)}{L^2}x^2\right] & \text{at} \quad a\le x\le L\end{cases}$$

3. Confirm that $M_z^e(x)=EIv^{e''}(x)$ is not a function of E. → OK for correspondence theorem.

4. Representative deflection: $\bar{x}=a$ (Hard to find max location, choose simplicity)

$$g_v(x)=\begin{cases}\dfrac{1}{6}\left[x^3-3ax^2-\dfrac{a^2(3L-2a)}{L^3}\left(x^3-3Lx^2\right)-\dfrac{3a^2(L-a)}{L^2}x^2\right] & \text{at} \quad 0\le x\le a\\ \dfrac{1}{6}\left[a^3-3a^2x-\dfrac{a^2(3L-2a)}{L^3}\left(x^3-3Lx^2\right)-\dfrac{3a^2(L-a)}{L^2}x^2\right] & \text{at} \quad a\le x\le L\end{cases}$$

$$g_v(\bar{x}=a)=-\frac{a^3(L-a)^3}{3L^3},\qquad \tilde{v}^e=-\frac{Pa^3(L-a)^3}{3EIL^3}$$

5. Shape functions

$$f_v(x)=\begin{cases}-\dfrac{L^3}{2a^3(L-a)^3}\left[x^3-3ax^2-\dfrac{a^2(3L-2a)}{L^3}\left(x^3-3Lx^2\right)-\dfrac{3a^2(L-a)}{L^2}x^2\right]\\ -\dfrac{L^3}{2a^3(L-a)^3}\left[a^3-3a^2x-\dfrac{a^2(3L-2a)}{L^3}\left(x^3-3Lx^2\right)-\dfrac{3a^2(L-a)}{L^2}x^2\right]\end{cases}$$

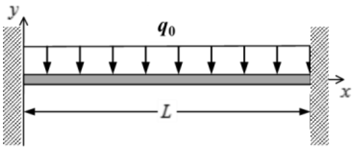
6. Generalized creep

$$\tilde{v}^{ve}=-\frac{a^3(L-a)^3}{3IL^3}\int_{0^-}^{t}J\left(t-\tau\right)\frac{dP(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{v}^{ve}=-\frac{a^3(L-a)^3}{3IL^3}J(t)P$$

7. Generalized relaxation

$$\tilde{q}^{ve}=-\frac{3IL^3}{a^3(L-a)^3}\int_{0^-}^{t}Y\left(t-\tau\right)\frac{d\tilde{v}(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{q}^{ve}=-\frac{3IL^3}{a^3(L-a)^3}Y(t)\tilde{v}$$

SI-4



$$v(x) = -\frac{q_0}{24EI}\left(x^4 - 2Lx^3 + L^2x^2\right)$$

1. Input

$$q(x) = q_0, \quad \tilde{q} = q_0$$

2. Elastic creep solution

$$v^e(x) = -\frac{q_0}{24EI}\left(x^4 - 2Lx^3 + L^2x^2\right)$$

3. Confirm that $M_z^e(x) = EIv^{e''}(x)$ is not a function of E. → OK for correspondence theorem.

4. Representative deflection: $\bar{x} = L/2$ (Choose the max deflection location)

$$g_v(x) = -\frac{1}{24}\left(x^4 - 2Lx^3 + L^2x^2\right)$$

$$g_v(\bar{x} = \frac{L}{2}) = -\frac{L^4}{384}, \qquad \tilde{v}^e = -\frac{q_0L^4}{384EI}$$

5. Shape functions

$$f_v(x) = \frac{16}{L^4}\left(x^4 - 2Lx^3 + L^2x^2\right)$$

6. Generalized creep

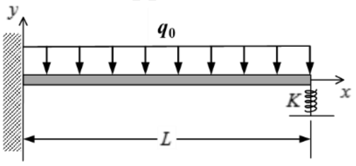
$$\tilde{v}^{ve} = -\frac{L^4}{384I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dq_0(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{v}^{ve} = -\frac{L^4}{384I}J(t)q_0$$

7. Generalized relaxation

$$\tilde{q}^{ve} = -\frac{384I}{L^4}\int_{0^-}^{t} Y\left(t-\tau\right)\frac{d\tilde{v}(\tau)}{d\tau}d\tau \quad \text{or} \quad \tilde{q}^{ve} = -\frac{384I}{L^4}Y(t)\tilde{v}$$

Special Statically-indeterminate Problems with Springs

General Approach



$$v^e(x) = -\frac{q_0}{24EI}\left[x^4 - 4Lx^3 + 6L^2x^2 + \frac{3KL^4}{2\left(3EI + KL^3\right)}\left(x^3 - 3Lx^2\right)\right]$$

1. Confirm that $M_z^e(x) = EIv^{e''}(x)$ is not a function of E.

$\rightarrow$ FALSE due to the red term

$\rightarrow$ Cannot apply the correspondence theorem directly.

$\rightarrow$ Use superposition principle from the start in order to apply correspondence theorem

$\rightarrow$ All SIK problems should be solved by the superposition method.

2. Use superposition method

$\rightarrow$ Obtain superposed elastic solutions: $v_A^e(x)$, $v_B^e(x)$, …

$\rightarrow$ Obtain superposed representative values: $\bar{x}$ must be at compatiblity condition

$$g_{v,A}(x),\ g_{v,B}(x),\ \ldots \quad | \quad g_{v,A}(\bar{x}),\ g_{v,B}(\bar{x}),\ \ldots \quad | \quad \tilde{v}_A^e,\ \tilde{v}_B^e,\ \ldots$$

$\rightarrow$ Obtain superposed shape functions: $f_{v,A}(x)$, $f_{v,B}(x)$, …

$\rightarrow$ Therefore,

Elastic: $v^e(x) = v_A^e(x) + v_A^e(x) + \ldots = \tilde{v}_A^e f_{v,A}(x) + \tilde{v}_B^e f_{v,B}(x) + \ldots$

Viscoelastic: $v^{ve}(x) = v_A^{ve}(x) + v_A^{ve}(x) + \ldots = \tilde{v}_A^{ve} f_{v,A}(x) + \tilde{v}_B^{ve} f_{v,B}(x) + \ldots$

$\rightarrow$ Determine general creep: $\tilde{v}_A^{ve}$, $\tilde{v}_B^{ve}$, …

Among these, there must be a superposed R_L which is unknown.

$\rightarrow$ Compatibility condition: $v^{ve}(\bar{x}) = \tilde{v}_A^{ve} f_{v,A}(\bar{x}) + \tilde{v}_B^{ve} f_{v,B}(\bar{x}) + \ldots = \tilde{v}_A^{ve} + \tilde{v}_B^{ve} + \ldots = -\dfrac{R_L}{K}$

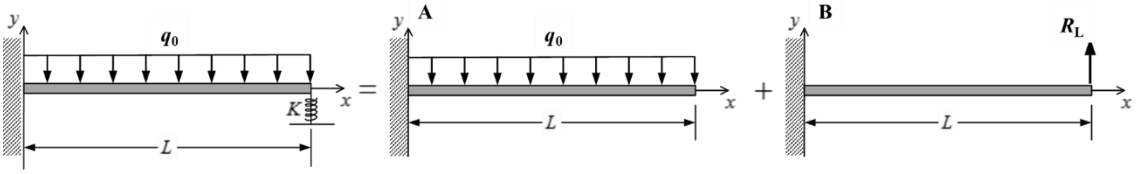
This is a very complex integral equation for R_L.

$\rightarrow$ Use Laplace Transformation and obtain an $\bar{R}_L(s)$ equation with $\bar{J}(s)$

$\rightarrow$ When we know the exact model, we can determine $R_L(t)$.

SIK-1

Superposition method (A: SD-2, B: SD-1 with $a = L$ and $P = -R_L$)



1. Input

$$q_A(x) = q_0, \quad \tilde{q}_A = q_0$$
$$q_B(x) = -R_L\delta(x-L), \quad \tilde{q}_B = -R_L$$

2. Elastic creep solutions

$$v_A^e(x) = -\frac{q_0}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$
$$v_B^e(x) = \frac{-R_L}{6EI}x^2\left(x-3L\right)$$

3. Representative deflection: $\bar{x} = L$ (Choose the location for compatiblity condition)

$$g_{v,A}(x) = -\frac{1}{24}\left(x^4 - 4Lx^3 + 6L^2x^2\right), \qquad g_{v,A}(\bar{x} = L) = -\frac{L^4}{8}, \qquad \tilde{v}_A^e = -\frac{q_0L^4}{8EI}$$
$$g_{v,B}(x) = \frac{1}{6}x^2\left(x-3L\right), \qquad g_{v,B}(\bar{x} = L) = -\frac{L^3}{3}, \qquad \tilde{v}_B^e = \frac{R_LL^3}{3EI}$$

4. Shape functions

$$f_{v,A}(x) = \frac{1}{3L^4}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$
$$f_{v,B}(x) = -\frac{1}{2L^3}x^2\left(x-3L\right)$$

5. Generalized creep

$$\tilde{v}_A^{ve} = -\frac{L^4}{8I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dq_0(\tau)}{d\tau}d\tau$$
$$\tilde{v}_B^{ve} = +\frac{L^3}{3I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dR_L(\tau)}{d\tau}d\tau$$

R_L is still unknown!!

6. Compatibility condition: $v(L) = -R_L/K$ for both elastic/viscoelastic cases

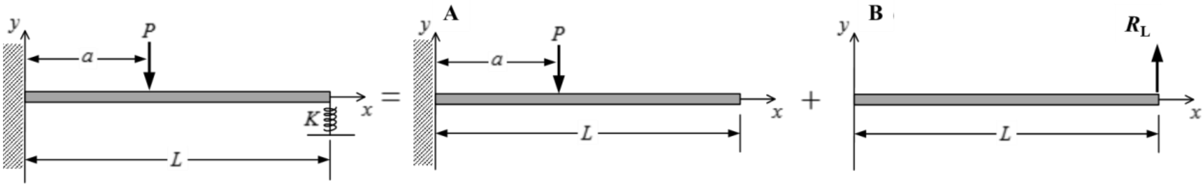
$$-\frac{L^4}{8I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dq_0(\tau)}{d\tau}d\tau + \frac{L^3}{3I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dR_L(\tau)}{d\tau}d\tau = -\frac{R_L}{K}$$

7. Laplace Transformation

$$\left(s\bar{J}(s) + \frac{3I}{KL^3}\right)\bar{R}_L(s) = \frac{3L}{8}s\bar{J}(s)\bar{q}_0(s)$$

SIK-2

<u>Superposition method</u> (A: SD-1, B: SD-1 with $a = L$ and $P = -R_L$)



1. Input

$$q_A(x) = P\delta(x-L),\ \tilde{q}_A = P \quad | \quad q_B(x) = -R_L\delta(x-L),\ \tilde{q}_B = -R_L$$

2. Elastic creep solutions

$$v_A^e(x) = \begin{cases} \dfrac{P}{6EI}x^2(x-3a) & \text{at} \quad 0 \le x \le a \\ \dfrac{P}{6EI}a^2(a-3x) & \text{at} \quad a \le x \le L \end{cases} \quad | \quad v_B^e(x) = \frac{-R_L}{6EI}x^2(x-3L)$$

3. Representative deflection: $\bar{x} = L$ (Choose the location for compatiblity condition)

$$g_{v,A}(x) = \begin{cases} \dfrac{1}{6}x^2(x-3a) & \text{at} \quad 0 \le x \le a \\ \dfrac{1}{6}a^2(a-3x) & \text{at} \quad a \le x \le L \end{cases} \quad | \quad g_{v,B}(x) = \frac{1}{6}x^2(x-3L)$$

$$g_{v,A}(\bar{x} = L) = \frac{a^2(a-3L)}{6}, \qquad \tilde{v}_A^e = \frac{Pa^2(a-3L)}{6EI}$$

$$g_{v,B}(\bar{x} = L) = -\frac{L^3}{3}, \qquad \tilde{v}_B^e = \frac{R_L L^3}{3EI}$$

4. Shape functions (self-derivation)

5. Generalized creep

$$\tilde{v}_A^{ve} = \frac{a^2(a-3L)}{6I}\int_{0^-}^{t} J(t-\tau)\frac{dP(\tau)}{d\tau}d\tau \quad | \quad \tilde{v}_B^{ve} = \frac{L^3}{3I}\int_{0^-}^{t} J(t-\tau)\frac{dR_L(\tau)}{d\tau}d\tau$$

6. Compatibility condition: $v(L) = -R_L/K$ for both elastic/viscoelastic cases

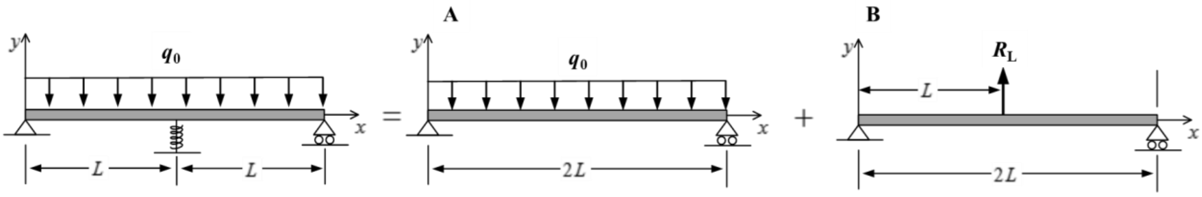
$$\frac{a^2(a-3L)}{6I}\int_{0^-}^{t} J(t-\tau)\frac{dP(\tau)}{d\tau}d\tau + \frac{L^3}{3I}\int_{0^-}^{t} J(t-\tau)\frac{dR_L(\tau)}{d\tau}d\tau = -\frac{R_L}{K}$$

7. Laplace Transformation

$$\left(s\bar{J}(s) + \frac{3I}{KL^3}\right)\bar{R}_L(s) = \frac{a^2(3L-a)}{2L^3}s\bar{J}(s)\bar{P}(s)$$

SIK-3

Superposition method (A: SD-4 with L=2L, B: SD-3 with $a = L$, L=2L and $P = -R_L$)



1. Input

$$q_A(x) = q_0,\ \tilde{q}_A = q_0 \quad | \quad q_B(x) = -R_L\delta(x-L),\ \tilde{q}_B = -R_L$$

2. Elastic creep solutions

$$v_A^e(x) = -\frac{q_0}{24EI}\left(x^4 - 4Lx^3 + 8L^3x\right)$$

$$v_B^e(x) = \begin{cases} \dfrac{-R_L}{12EI}\left(x^3 - 3L^2x\right) & \text{at} \quad 0 \le x \le L \\ \dfrac{-R_L}{12EI}\left(x^3 - 3L^2x - 2\left(x-L\right)^3\right) & \text{at} \quad L \le x \le 2L \end{cases}$$

3. Representative deflection: $\bar{x} = L$ (Choose the location for compatiblity condition)

$$g_{v,A}(x) = -\frac{1}{24}\left(x^4 - 4Lx^3 + 8L^3x\right)$$

$$g_{v,B}(x) = \begin{cases} \dfrac{1}{12}\left(x^3 - 3L^2x\right) & \text{at} \quad 0 \le x \le L \\ \dfrac{1}{12}\left(x^3 - 3L^2x - 2\left(x-L\right)^3\right) & \text{at} \quad L \le x \le 2L \end{cases}$$

$$g_{v,A}(\bar{x} = L) = -\frac{5L^4}{24},\ \tilde{v}_A^e = -\frac{5q_0L^4}{24EI} \quad | \quad g_{v,B}(\bar{x} = L) = -\frac{L^3}{6},\ \tilde{v}_B^e = \frac{R_LL^3}{6EI}$$

4. Shape functions (self-derivation)

5. Generalized creep

$$\tilde{v}_A^{ve} = -\frac{5L^4}{24I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dq_0(\tau)}{d\tau}d\tau \quad | \quad \tilde{v}_B^{ve} = \frac{L^3}{6I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dR_L(\tau)}{d\tau}d\tau$$

6. Compatibility condition: $v(L) = -R_L/K$ for both elastic/viscoelastic cases

$$-\frac{5L^4}{24I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dq_0(\tau)}{d\tau}d\tau + \frac{L^3}{6I}\int_{0^-}^{t} J\left(t-\tau\right)\frac{dR_L(\tau)}{d\tau}d\tau = -\frac{R_L}{K}$$

7. Laplace Transformation

$$\left(s\bar{J}(s) + \frac{6I}{KL^3}\right)\bar{R}_L(s) = \frac{5L}{4}s\bar{J}(s)\bar{q}_0(s)$$

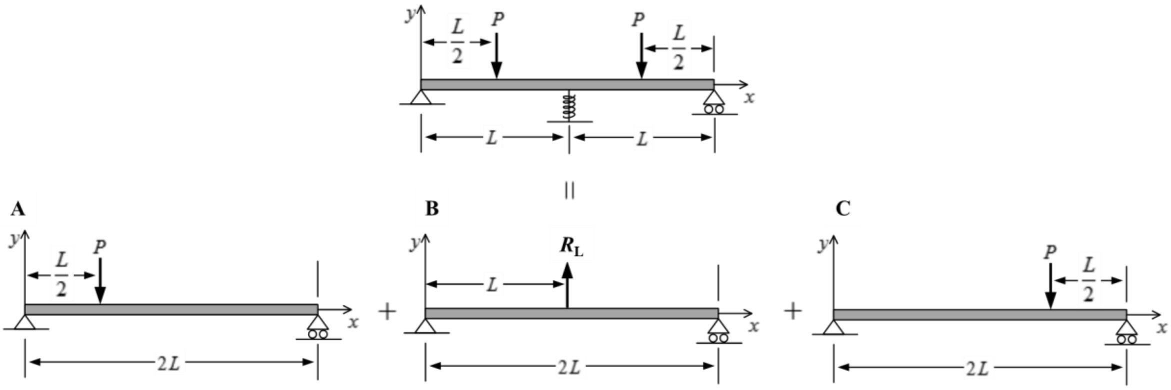
SIK-4

Superposition method

A: SD-3 with $a = L/2$ and L=2L

B: SD-3 with $a = L$, L=2L and $P = -R_L$

C: SD-3 with $a = 3L/2$ and L=2L



1. Input

$$q_A(x) = P\delta\left(x - \frac{L}{2}\right), \qquad \tilde{q}_A = P$$

$$q_B(x) = -R_L\delta\left(x - L\right), \quad \tilde{q}_B = -R_L$$

$$q_C(x) = P\delta\left(x - \frac{3L}{2}\right), \quad \tilde{q}_A = P$$

2. Elastic creep solutions

$$v_A^e(x) = \begin{cases} \dfrac{P}{32EI}\left(4x^3 - 7x\right) & \text{at} \quad 0 \le x \le \dfrac{L}{2} \\ \dfrac{P}{32EI}\left(4x^3 - 7x - \dfrac{16}{3}\left(x - \dfrac{L}{2}\right)^3\right) & \text{at} \quad \dfrac{L}{2} \le x \le 2L \end{cases}$$

$$v_B^e(x) = \begin{cases} \dfrac{-R_L}{12EI}\left(x^3 - 3L^2x\right) & \text{at} \quad 0 \le x \le L \\ \dfrac{-R_L}{12EI}\left(x^3 - 3L^2x - 2\left(x - L\right)^3\right) & \text{at} \quad L \le x \le 2L \end{cases}$$

$$v_C^e(x) = \begin{cases} \dfrac{P}{96EI}\left(4x^3 - 15x\right) & \text{at} \quad 0 \le x \le \dfrac{3L}{2} \\ \dfrac{P}{96EI}\left(4x^3 - 15x - 16\left(x - \dfrac{3L}{2}\right)^3\right) & \text{at} \quad \dfrac{3L}{2} \le x \le 2L \end{cases}$$

3. Representative deflection: $\bar{x} = L$ (Choose the location for compatiblity condition)

$$g_{v,A}(\bar{x} = L) = -\frac{11L^3}{96}, \qquad \tilde{v}_A^e = -\frac{11PL^3}{96EI}$$

$$g_{v,B}(\bar{x} = L) = -\frac{L^3}{6}, \qquad \tilde{v}_B^e = \frac{R_L L^3}{6EI}$$

$$g_{v,C}(\bar{x} = L) = -\frac{11L^3}{96}, \qquad \tilde{v}_C^e = -\frac{11PL^3}{96EI}$$

4. Shape functions (self-derivation)

5. Generalized creep

$$\tilde{v}_A^{ve} = -\frac{11L^3}{96I}\int_{0^-}^{t} J(t-\tau)\frac{dP(\tau)}{d\tau}d\tau$$

$$\tilde{v}_B^{ve} = \frac{L^3}{6I}\int_{0^-}^{t} J(t-\tau)\frac{dR_L(\tau)}{d\tau}d\tau$$

$$\tilde{v}_C^{ve} = -\frac{11L^3}{96I}\int_{0^-}^{t} J(t-\tau)\frac{dP(\tau)}{d\tau}d\tau$$

6. Compatibility condition: $v(L) = -R_L/K$ for both elastic/viscoelastic cases

$$-\frac{11L^3}{48I}\int_{0^-}^{t} J(t-\tau)\frac{dq_0(\tau)}{d\tau}d\tau + \frac{L^3}{6I}\int_{0^-}^{t} J(t-\tau)\frac{dR_L(\tau)}{d\tau}d\tau = -\frac{R_L}{K}$$

7. Laplace Transformation

$$\left(s\bar{J}(s) + \frac{6I}{KL^3}\right)\bar{R}_L(s) = \frac{11}{8}s\bar{J}(s)\bar{P}(s)$$